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MA111 - Engineering Mathematics - II
Problem Sheet - 8

Second Order Homogeneous Linear ODE

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1. Use the reduction of order to find a second solution for $x^2y'' - xy' + y = 0$ given that $y_1(x) = x$ is a solution. Also, solve the initial value problem with initial conditions $y(1) = 2$ and $y'(1) = 1$.
ANS: $y(x) = 2x - x\log x$

2. Solve by reducing the order $xy'' - (2x-1)y' + (x-1)y = 0$ given that $y_1(x) = e^x$ is a solution.
ANS: $y(x) = C_1e^x \log x + C_2e^x$

3. Solve by reducing the order $x^2y'' + xy' - y = 0$ given that $y_1(x) = x + \frac{1}{x}$ is a solution. **ANS:** $y(x) = C_2(x + \frac{1}{x}) + C_1(\frac{1}{x})$

4. Solve by reducing the order $x^2y'' - 5xy' + 9y = 0$ given that $y_1(x) = x^3$ is a solution.
ANS: $y(x) = C_1x^3 + C_2x^3 \log x$

5. Find the curve through the origin in the xy-plane which satisfies $y'' = 2y'$ and whose tangent at the origin has slope 1. **ANS:** $y(x) = C_1e^{2x} + C_2$

6. Verify that the given functions are linearly independent and form a basis of solutions of the given ODE. Solve the Initial value problem.
   (a) $y'' + 2y' + 2y = 0, y(0) = 0, y'(0) = 15, y_1(x) = e^{-x}\cos x, y_2(x) = e^{-x}\sin x.$
   (b) $x^2y'' - xy' + y = 0, y(1) = 4.3,\ y'(1) = 0.5, y_1(x) = x,\ y_2(x) = x\log x.$
   (c) $(x\sin x + \cos x)y'' - x\cos x - y' + y\cos x, y_1(x) = x,\ y_2(x) = \cos x.$

7. Let $f_1(x) = x$ and $f_2(x) = |x|$ on $[-1, 1]$. Prove or disprove $f_1(x)$ and $f_2(x)$ are linearly independent.

8. Let $f_1(x) = 1 - \cos^2 t$ and $f_2(x) = 2\sin^2 t$ on $\mathbb{R}$. Prove or disprove $f_1(x)$ and $f_2(x)$ are linearly independent.

9. Solve the following differential equations
   (a) $y'' + 2y' + 5y = 0$ (**ANS**: $y(x) = e^{-x}(\cos 4x + C_2 \sin 4x)$)
   (b) $y'' + 4y = 0,\ y(0) = 2$ and $y'(0) = 0$. (**ANS**: $y(x) = 2\cos 2x$)
   (c) $y'' + 2py' + (p^2 + q^2)y = 0$. (**ANS**: $y(x) = e^{-px}(C_1 \cos qx + C_2 \sin qx)$
   (d) $ly'' + ky = 0,\ y(0) = y_0$ and $y'(0) = 0$, where $k, l \in \mathbb{R}$ and $l \neq 0$.
   (**ANS**: $y(x) = y_0 \cos\left(x\sqrt{\frac{k}{l}}\right)$)
   (e) $y'' - 6y' + 8y = 0,\ y(0) = -2,\ y'(0) = 6$ (**ANS:** $y(x) = -7e^{2x} + 5e^{4x}$)
   (f) $y'' - 6y' + 13y = 0,\ y(0) = 0,\ y'(0) = 10$. (**ANS:** $y(x) = 5e^{3x}\sin(2x)$)
   (g) $y'' - 4y + 5y = 0$. (**ANS:** $y(x) = e^{2x}(C_1 \cos x + C_2 \sin x)$)

(h) $y'' + 25y = 0$. (**ANS:** $y(x) = C_1 \cos 5x + C_2 \sin 5x$)

10. Solve the following differential equations

   (a) $x^2y'' + xy' - 4y = 0$. (**ANS**: $y(x) = C_1x^2 + C_2x^{-2}$)
   (b) $x^2y'' - 3xy' + 4y = 0$. (**ANS**: $y(x) = (C_1 + C_2 \log x)x^2$)
   (c) $x^2y'' + y = 0$.(**ANS**: $y(x) = x^{\frac{1}{2}}(C_1 \cos(\frac{\sqrt{3}}{2} \log x) + C_2 \sin(\frac{\sqrt{3}}{2} \log x)))$
   (d) $x^2y'' + 7xy' + 13y = 0$. (**ANS:** $y(x) = x^{-3}(C_1 \cos(2 \log x) + C_2 \sin(2 \log x)))$
   (e) $x^2y'' - 6xy' - 18y = 0$. (**ANS**: $y(x) = C_1x^{-2} + C_2x^9$)
   (f) $x^2y'' - xy' - 8y = 0$. (**ANS:** $y(x) = (C_1 + C_2 \log x)\sqrt{x}$)
   (g) $4x^2y'' + y = 0$. (**ANS:** $y(x) = C_1x^4 + \frac{C_2}{x^2}$)

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